NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

ROUTING A HIGH VALUE UNIT FOR OPTIMIZED
MISSILE DEFENSE IN COASTAL WATERS

by

John M. Baker

March 2008

Thesis Advisor: R. Kevin Wood
Second Reader: Johannes O. Royset

Approved for public release; distribution is unlimited

THIS PAGE INTENTIONALLY LEFT BLANK

1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** March 2008	**3. REPORT TYPE AND DATES COVERED** Master's Thesis
4. TITLE AND SUBTITLE Routing a High Value Unit for Optimized Missile Defense in Coastal Waters		**5. FUNDING NUMBERS**
6. AUTHOR(S) John M. Baker		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000		**8. PERFORMING ORGANIZATION REPORT NUMBER**
9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A		**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.	
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited	**12b. DISTRIBUTION CODE**

13. ABSTRACT (maximum 200 words)

This thesis addresses the problem of routing a navy Carrier Strike Group (CSG), or other groups of ships, through a maritime chokepoint that presents one or more shore-based missile threats. The goal is to identify a path that minimizes risk to the CSG's HVU (High Value Unit, i.e., the CSG's aircraft carrier). The HVU's escort ships are assigned optimal positions relative to the HVU during the transit to maximize the overall probability of avoiding and/or defeating attacks. The problem is formulated and solved as a maximum-reliability path problem in a network: The operating environment is discretized into a grid of nodes that represents potential waypoints, escort formations, and travel directions; arcs define allowable transitions between nodes. An arc parameter represents the probability of successfully transiting between two adjacent nodes, computed as a function of formation, direction of travel, threat, and line-of-sight visibility between any threats and the CSG. A test scenario, with a node spacing of 2.5 nautical miles, approximates the Strait of Hormuz. The model solves in a fraction of a second on a personal computer. Results show that the CSG typically places escorts ahead of the HVU, and always between the HVU and the closest threat.

14. SUBJECT TERMS Carrier Strike Group, Maximum-Reliability Path, Shortest-Path Algorithm.			**15. NUMBER OF PAGES** 57
			16. PRICE CODE
17. SECURITY CLASSIFICATION OF REPORT Unclassified	**18. SECURITY CLASSIFICATION OF THIS PAGE** Unclassified	**19. SECURITY CLASSIFICATION OF ABSTRACT** Unclassified	**20. LIMITATION OF ABSTRACT** UU

THIS PAGE INTENTIONALLY LEFT BLANK

ROUTING A HIGH VALUE UNIT FOR OPTIMIZED MISSILE DEFENSE IN COASTAL WATERS

John M. Baker
Lieutenant, United States Navy
B.S., United States Naval Academy, 2000

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
March 2008

Author: John M. Baker

Approved by: Dr. R. Kevin Wood
                    Thesis Advisor

                    Dr. Johannes O. Royset
                    Second Reader

                    Dr. James N. Eagle
                    Chairman, Department of Operations Research

THIS PAGE INTENTIONALLY LEFT BLANK

ABSTRACT

This thesis addresses the problem of routing a navy Carrier Strike Group (CSG), or other groups of ships, through a maritime chokepoint that presents one or more shore-based missile threats. The goal is to identify a path that minimizes risk to the CSG's HVU (High Value Unit, i.e., the CSG's aircraft carrier). The HVU's escort ships are assigned optimal positions relative to the HVU during the transit to maximize the overall probability of avoiding and/or defeating attacks. The problem is formulated and solved as a maximum-reliability path problem in a network: The operating environment is discretized into a grid of nodes that represents potential waypoints, escort formations, and travel directions; arcs define allowable transitions between nodes. An arc parameter represents the probability of successfully transiting between two adjacent nodes, computed as a function of formation, direction of travel, threat, and line-of-sight visibility between any threats and the CSG. A test scenario, with a node spacing of 2.5 nautical miles, approximates the Strait of Hormuz. The model solves in a fraction of a second on a personal computer. Results show that the CSG typically places escorts ahead of the HVU, and always between the HVU and the closest threat.

THIS PAGE INTENTIONALLY LEFT BLANK

TABLE OF CONTENTS

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF FIGURES

LIST OF TABLES

THIS PAGE INTENTIONALLY LEFT BLANK

EXECUTIVE SUMMARY

This thesis develops a model and algorithm to calculate the safest route possible for a navy Carrier Strike Group (CSG) that must transit a maritime chokepoint while subject to one or more shore-based missile threats. The notional CSG consists of a high-value unit (HVU), i.e., the strike group's aircraft carrier, and its escort ships. The algorithm identifies the optimal route for the HVU, which also includes the best relative stations for each of its missile-defense escorts during each leg of the transit. Specific sets of escort stations are enumerated and referred to herein as "configurations."

The Naval Research Laboratory (NRL) has conducted work related to the topic of this thesis. Specifically, NRL has developed an algorithm to assess Anti-Air Warfare (AAW) engagement capabilities for U.S. Navy ships based on radio-frequency (RF) propagation of threat radar. This algorithm estimates an HVU's probability of survival at a given location based on threat radar capabilities and engagement geometry for a single escort against a single threat. The goal of this thesis is to develop an algorithm that will use those probabilities to automatically find a minimum-risk route for a group of ships, such as a CSG.

The problem is formulated and solved using a maximum-reliability path model (XRP), whose solution yields an optimal transit path for the CSG through a notional sea space called the "area of transit" (AT). XRP discretizes the "area of transit" (AT) into a network of vertices and edges representing potential waypoints and movement between waypoints, respectively. Vertices are then "expanded" to represent potential waypoints, escort configurations, and directions of travel; expanded edges then connect waypoints, but also represent transitions between configurations and directions of travel. Independence between edges is assumed, and an edge parameter represents the probability of successfully transiting between two adjacent waypoints, along with making any changes in configuration and direction of travel. The solution algorithm is a variant of a shortest-path algorithm. It operates implicitly on the expanded network and maximizes the product of success probabilities across the edges in a path. Success

probabilities are computed as a function of configuration; direction of travel; threats; and distance and line-of-sight visibility to those threats.

The test scenario for this thesis defines an approximation of the Strait of Hormuz as the CSG's AT. The network is constructed to cover the entire AT and its bordering entry and exit points in the Gulf of Oman and Arabian Gulf, respectively. Five shore-based missile sites threaten the AT, and four escorts defend the HVU. The CSG can transition between 16 different configurations and eight directions of travel at every node; however, only certain configuration transitions are allowed because of transit-speed constraints. XRP calculates and then writes to a text file the maximum-reliability route through the AT by vertex location, configuration, and direction of travel. Results also display the computed cumulative probabilities of successful passage at each waypoint in the route.

A number of simplifying assumptions have been made, but the assumptions are reasonable for a prototype, and the optimal routes identified by the algorithm appear to be good. Also, in order to keep this research unclassified, no real threat data, such as probabilities of kill, are used.

The XRP solution algorithm is written in C++ and runs on a desktop PC; an optimal solution is identified in a fraction of a second. Results show that based on transit geometry in the AT, the CSG keeps threats at a maximum distance by staying close to "untransitable areas" such as shoal water and land. The CSG also relies primarily on configurations that place escorts ahead of the HVU, and always between the HVU and the closest threat.

ACKNOWLEDGMENTS

I would like to thank my advisor, Professor Kevin Wood, for his outstanding guidance, patience, and help with C++. I would also like to thank my second reader, Professor Johannes Royset, for his initial model implementation and guidance.

THIS PAGE INTENTIONALLY LEFT BLANK

I. INTRODUCTION

Safe transit through maritime chokepoints is critical for the United States Navy to fulfill its mission of maintaining freedom of the seas. Command of littoral (coastal) waters has become a priority in recent years, and access to many littoral regions of the world requires unhindered passage through maritime chokepoints. This thesis develops a mission-planning aid in support of a hypothetical navy Carrier Strike Group (CSG) that needs to make a transit through such a chokepoint, while subject to one or more shore-based cruise-missile threats. Specifically, given a CSG consisting of one HVU (High-Value Unit, i.e., an aircraft carrier) and several escorts that defend the HVU, the planning aid identifies a path, and escort formations along that path, that maximize the probability that the HVU makes the transit safely.

A. BACKGROUND

This problem was introduced to us by Mr. Ian Will at the Naval Research Laboratory (NRL) in Washington, DC. NRL is developing an algorithm for assessing Anti-Air Warfare (AAW) engagement capabilities for ships based on radio-frequency (RF) propagation of threat radar (Sjoberg et al., 2006). This algorithm will estimate an HVU's probability of survival at a given location based on threat radar capabilities and engagement geometry for a single escort. The goal of this thesis is to develop an algorithm that can use those probabilities to automatically find a minimum-risk path for a CSG.

At this time, NRL's simulation techniques are too slow to provide all the probabilities needed for the path-finding algorithm in a reasonable amount of time, that is, fast enough for the path-finding algorithm to be used as an operational tool (Sjoberg et al., 2006). Another concern is properly accounting for AAW capabilities when multiple ships are tasked with the HVU's defense. NRL is also looking for ways to speed up computations and to handle the multiple-defender case accurately. In the meantime, this

1

thesis will assume that the probability of successful defense for a single escort is available, and will combine probabilities for multiple escorts under the assumption of independence.

NRL uses the term "defender" to mean any AAW-capable ship. The HVU is the "defender" in this thesis, and "escort" means any AAW-capable ship in the CSG. AAW-capable ships comprise *Ticonderoga*-class cruisers (CGs) and *Arleigh Burke*-class destroyers (DDGs) with the AEGIS combat weapons system. The model developed in this thesis computes the defender's approximate probability of successful defense and passage between waypoints in the AT; the threat to each segment of the path comes from shore-based anti-ship cruise-missile (ASCM) sites. Probability of survival can be computed assuming the HVU has any number of escorts, although all tests are restricted to four. A variant on a shortest-path network model, a "maximum-reliability path model" (XRP), then defines a path through the chokepoint that minimizes total risk for the HVU, and the optimal configuration of escorts for each path. A variant on a shortest-path algorithm solves XRP model instances.

The XRP model applies to any maritime chokepoint, although the Strait of Hormuz provides the test scenario in this thesis. Other examples of maritime chokepoints include the Strait of Gibraltar between Spain and Morocco, and the Strait of Malacca between the Indonesian island of Sumatra and the Malaysian peninsula.

B.   LITERATURE REVIEW

1.   Previous Work

Naval Postgraduate School Professors Johannes Royset and Kevin Wood have collaborated with Ian Will to develop a working example of a minimum-risk path algorithm. Royset and Wood (2007) have modified the technology used to find minimum-risk paths for aircraft to help solve the CSG-routing problem. The model uses a bearing-based metric for predicting defensive capabilities for a single escort, and the positions (bearings) of a set of escorts, called a "configuration" here (but often referred to as an "escort formation" in the U.S. Navy) are combined, under independence, to specify

the defensive capability of the CSG. For instance, a standard configuration might put two of the CSG's escorts in front of the HVU, one on each side of the bow, and put two escorts behind the HVU, one on each quarter. But, if the only possible threats lie directly in front of the HVU, the preferred configuration might put all four escorts in positions ahead of the HVU, configured around the HVU's port and starboard bow, and possibly on the port and starboard beam.

For a transiting CSG, Royset and Wood construct a "physical network" $G = (V, E)$ with vertices $v \in V$ representing waypoints in the ocean, that are connected by edges, $(u, v) \in E$ that define potential movements (ignoring escort configurations). Each vertex in G is then "expanded" into an abstract network $H = (N, A)$ consisting of nodes, $i \in N$, connected by arcs $k = (i, j) \in A$. A node $i = \langle v, f \rangle$ corresponds to a vertex v and an escort configuration f. Thus, arcs allow transitions between physical locations and configurations, although some possible transitions are disallowed because of physical constraints. A path in the expanded network from some node $\langle s, f \rangle$ to some node $\langle t, f' \rangle$ corresponds to a path in the physical network from start vertex s where the CSG enters the AT, to an end vertex t where it exits the AT, along with the various configurations and directions of travel that the CSG uses along the path.

If we let ϕ_k denote the probability of successfully transiting arc k, i.e., with no successful missile strikes, then the probability of a successful transit along a path of arcs A_{Path} is

$$\prod_{k \in A_{Path}} \phi_k \,, \tag{1}$$

assuming independence across arcs. This product is maximized by minimizing

$$\sum_{k \in A_{Path}} -\log \phi_k \,. \tag{2}$$

Thus, a minimum-risk path can be computed using standard shortest-path calculations if the "length" of arc k is defined as $-\log \phi_k$ (e.g., Royset et al. 2008). Transitions

between configurations are limited in order to model speed constraints: the CSG's escorts will have specified stationing-speed restrictions and will be unable to transition into specific sectors relative to the HVU. For example, given the CSG's speed of advance, maneuvering from dead astern of the HVU to directly ahead may be infeasible.

The model developed by Royset and Wood (2007) is related to the constrained shortest-path model (CSP) in Royset et al. (2008) used for routing military aircraft. The latter paper uses a two- or three-dimensional network to model aircraft transiting two- or three-dimensional space, respectively. The solution to this model routes aircraft from origin to target to minimize the risk of destruction from ground-based surface-to-air missile sites (and possibly other threats), while placing constraints on fuel consumption and/or flight time.

This thesis uses a model that looks much like the minimum-risk CSP model for aircraft routing, except that standardized probabilities are computed much differently and only a two-dimensional network is relevant. Factors such as flight time and/or fuel constraints, which are critical to aircraft routing, are irrelevant to this particular ship-routing problem. Critical to this problem is the configuration of escort ships in the CSG, because those configurations affect the risk imposed on the HVU from threat missiles.

One innovation in this research, versus Royset and Wood, is that the expanded network adds direction of travel in order to represent probability of survival more accurately. We use a maximum-reliability path model (XRP), and solution algorithm, to find a minimum-risk path. The algorithm operates on an implicit representation of the expanded network instead of an explicit one, and directly maximizes (1), rather than minimizing (2). Chapter III discusses XRP in more detail.

Nothing like the aircraft-routing model of Royset et al. exists for naval surface applications. This thesis attempts to fill the gap.

2. Navy Tactical Publications

Two Navy Tactical Memorandums (TMs) are relevant to this research. Written by the Surface Warfare Development Group (SWDG), these TMs are intended for use by

task force commanders, strike group staffs, ships' commanding officers and their respective Air Defense (AD) training teams. Both TMs are classified at the secret level; the following three paragraphs describe general features of each at the unclassified level.

TM 3-01.4-05 (2005), entitled *AEGIS Overland Tactics* (U), provides performance capabilities and limitations of the Aegis Weapons System (AWS) in an overland environment, and identifies controllable and uncontrollable factors in each phase of an Aegis ship's engagement sequence. It also includes terrain-analysis charts for the Arabian Gulf, Strait of Hormuz, and other littoral regions (TM 3-01.4-05 2005, p. EX-1).

TM 3-01.5-01, entitled *Air Defense Ship Stationing in a Littoral Antiship Missile Environment* (U), addresses a widely held concern: how to best station combatant ships with respect to high-value naval and commercial vessels to maximize protection against a known threat in a littoral region. Historically, Aegis ships have been employed in an open-ocean environment, unencumbered by close proximity to land. The ships have used the concept of "defense-in-depth," which includes the capability to engage an air threat at a distance of 100 nautical miles (nm) in the open ocean. But in a littoral environment, the threat may originate at a distance of 20 nm or less. Rapid threat detection and engagement become critical for ship survivability in this case.

Although Aegis ships' sensors have been improved in recent years, little has been done to investigate the effectiveness of their weapons in close proximity to land. Because of the reduced distance and time factors experienced in littoral regions and chokepoints, proper AD planning is even more critical than before (TM 3-01.5-01, 2003, p. EX-1). TM 3-01.5-01 provides model and simulation data to assist AD planners in developing and improving AD capabilities for their ships.

Though this thesis differs in its approach, its goal is the same as the TMs described above: improve AD planning and make surface ships more effective in missile-threat situations in a littoral environment.

C.   PROBLEM STATEMENT AND SOLUTION-METHOD OUTLINE

The maximum-reliability path for a CSG will describe a physical route through a chokepoint, along with appropriate escort configurations for each leg of the path; the path will maximize the HVU's probability of survival (i.e., minimizes "risk") when threatened by shore-based Anti-Ship Cruise Missiles (ASCMs). Path risk through a chokepoint depends on factors such as escort-ship configuration relative to the HVU and threat locations. The fact that a configuration is relative to the HVU means that direction of travel will also be tracked in the XRP model. We let $d \in D$ indicate direction of travel in the expanded network, and add d to Royset and Wood's formulation (Royset and Wood 2007) so that a node is represented by $i = \langle v, f, d \rangle$.

Several assumptions simplify the calculation of ϕ_k^A, the probability of successfully transiting arc k. First, one specific type of shore-based threat ASCM is used: the C-802, which was developed in the mid-1980s by China (Jane's Naval Weapon Systems 2006, pp. 287-289). In order to keep this research unclassified, the C-802 is assumed to have a nominal threat range of 25 nm. We assume a precomputed probability of successful defense for a single escort in a given configuration and direction of travel at each vertex in the network assuming the threat remains constant for a standard distance equal to the maximum physical length of any arc, and use those values to compute ϕ_i^N, the probability of a successful defense at each node $i = \langle v, f, d \rangle$ in the expanded network (i.e., the probability of successful defense against all threats at each vertex, for each possible multi-escort configuration and direction of travel, assuming that the threat remains constant over a standard distance). Finally, we assume that the probability of successfully transiting from $i \in N$ to $j \in N$ along arc (i, j)—this is ϕ_k^A where $k = (i, j)$—is an average of ϕ_i and ϕ_j, weighted by the physical length of the arc. This is a crude assumption, but should suffice for demonstration purposes.

Consider a CSG consisting of an aircraft carrier and four AAW-capable escort ships. The CSG must safely transit the Strait of Hormuz to gain entry to the Arabian Gulf and begin its mission tasking in the U.S. 5th Fleet Area of Responsibility (AOR). As the

chokepoint region in this case, the Strait of Hormuz is referred to as the Area of Transit (AT), and the Arabian Gulf is referred to as the AOR (Figure 1).

The best configuration during a transit is complicated and can change as the transit progresses. For instance, if escorts are positioned directly between a threat missile's path and the HVU, the probability of the missile hitting the HVU is low. And, based on transit geometry in the AT, we see that (a) if there are equal threats on either side of the CSG, then the best position for four escorts would be two on the HVU's port beam, and two on the HVU's starboard beam, (b) as the CSG moves into a threat area, any threats are probably positioned in the direction of travel, so, initially, the escorts should lead the HVU, and (c) as the CSG leaves the threat area, any threat is probably astern of the HVU, so the escorts should trail the HVU.

The remainder of the thesis is outlined as follows. Chapter II introduces the XRP model for transiting a chokepoint; Chapter III describes how to solve XRP using an "implicit maximum-reliability path algorithm" (IXRPA); Chapter IV describes a test scenario and presents corresponding computational results; and Chapter V presents a summary, conclusions, and suggestions for further research.

Figure 1. Entrance to the Arabian Gulf and Strait of Hormuz (source: Microsoft MapPoint).

II. A MAXIMUM-RELIABILITY PATH MODEL FOR SHIP ROUTING

This chapter begins by discussing a generic maximum-reliability path network model (XRP), and defines the physical network used to represent the AT. We then describe the expanded network on which XRP is defined, and define escort configurations, direction on travel, and sector representation in the model. The chapter concludes with a description of the probabilities of successful defense used in our formulation.

A. MAXIMUM-RELIABILITY PATHS

This research models the CSG-routing problem as the problem of finding a minimum-risk path in a network. This is, essentially, a "maximum-reliability path." A generic maximum-reliability path problem is closely related to the formulation of a shortest-path problem (Bobzin 2005, p.107). Standard shortest-path calculations seek to minimize the sum of arc "lengths" in a network. The formulation of a maximum-reliability path, however, treats arcs as a measure of reliability, and the reliability of a directed path P is given by the product (rather than the sum) of the reliability of arcs in the path (Ahuja et al., 1993, p. 130). The maximum-reliability path problem, then, is to identify a directed path in the network from the source vertex s to the terminal vertex t that maximizes reliability, i.e., probability of successful defense of the HVU.

B. NETWORK CONSTRUCTION

1. Physical Network

The path-finding algorithm uses an "expanded network" to identify transit paths over a hypothetical AT (Figure 2). That network is based on a *physical network* $G = (V, E)$ in which the surface of the ocean in the AT is discretized into a set of vertices $v \in V$ representing potential waypoints in two-dimensional space; these vertices are connected by directed edges $(u, v) \in E$ to represent potential transit segments for the

CSG between distinct vertices $u, v \in V$. The CSG will transit from waypoint to waypoint along, and in the direction of, the specified edges (Carlyle et al. 2008).

In this thesis, vertices are positioned in a grid with spacing in both coordinate axes of 2.5 nm (5000 yards). The example network covers an area 125 nm wide by 62.5 nm high: 51 vertices in the east-west, or x-coordinate direction, and 26 vertices in the north-south, or y-coordinate direction. This contains land and shoal water that a CSG cannot traverse: vertices are created in this case, but not connected by edges.

Edges $(u, v) \in E$ connect vertices, except as noted above. Each vertex that corresponds to "traversable water" is connected to its nearest neighbors, assuming they exist in the grid and are "traversable," in the compass bearings 000, 045, 090, 135, 180, 225, 270, and 315 degrees. Montes (2005) suggests this spatial adjacency; see Figure 3. Each edge $(u, v) \in E$ has a corresponding "physical length" that represents the geometric distance in nautical miles between edges u and v: An edge directed north, south, east or west has a length of 2.5 nm while an edge directed northwest, northeast, southwest or southeast has a length of $\sqrt{12.5} \approx 3.5$ nm.

A notional representation of the AT is shown in Figure 2 (the dimensions and placements are inexact). Vertices are placed at the corners of the square boxes (there are no nodes at the intersections of the diagonal lines), and the heavy lines represent traversable arcs. For simplicity, threat ASCM sites are located at network nodes on land, close to shore. They are denoted as black dots in the figure.
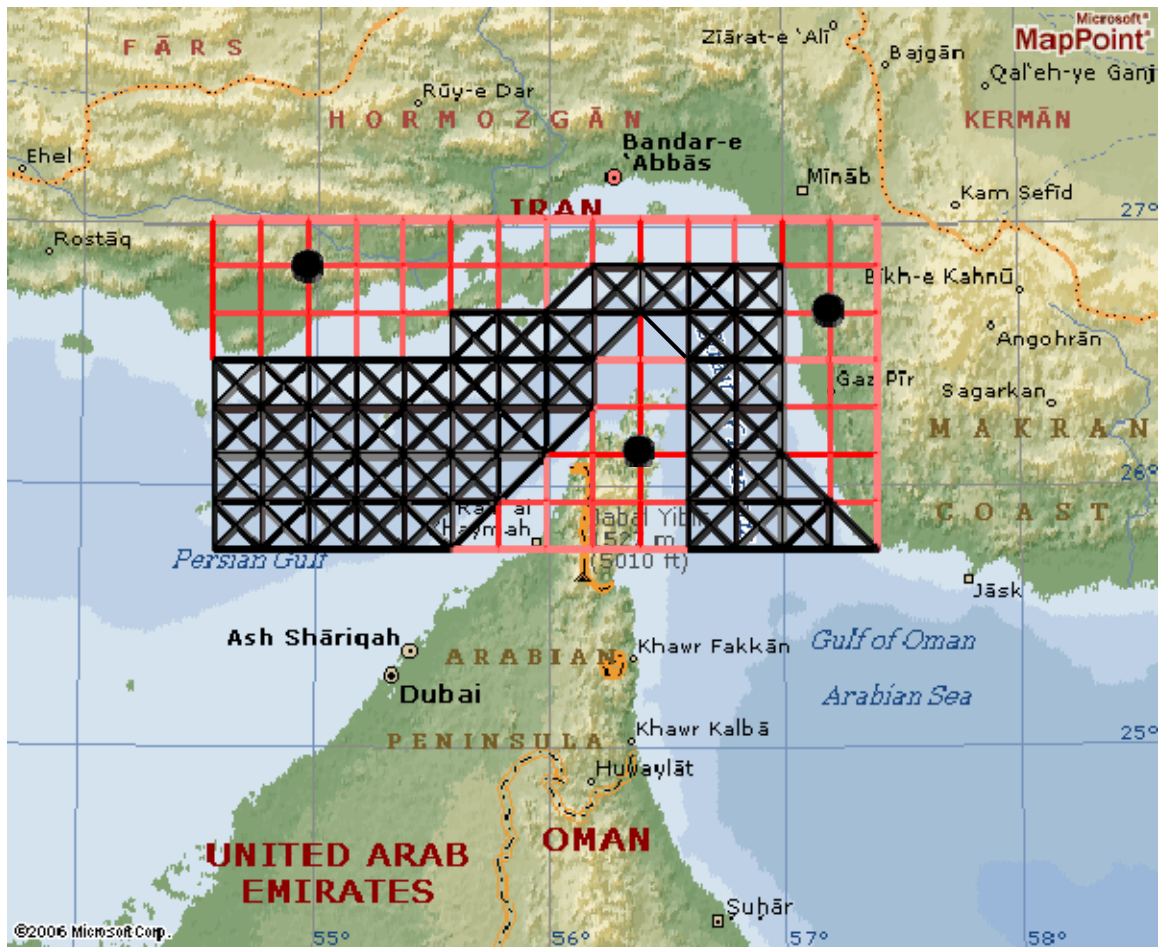
Figure 2.   Notional representation of the AT (dimensions and placements not exact). Vertices lie at the corners of the square boxes (there are no vertices at the intersections of the diagonal edges), and the heavy lines represent traversable edges.   Threat ASCM sites are shown as three large black dots on three "untraversable vertices," but, note that the later computational study uses different threat locations. (Source: Microsoft MapPoint).

11
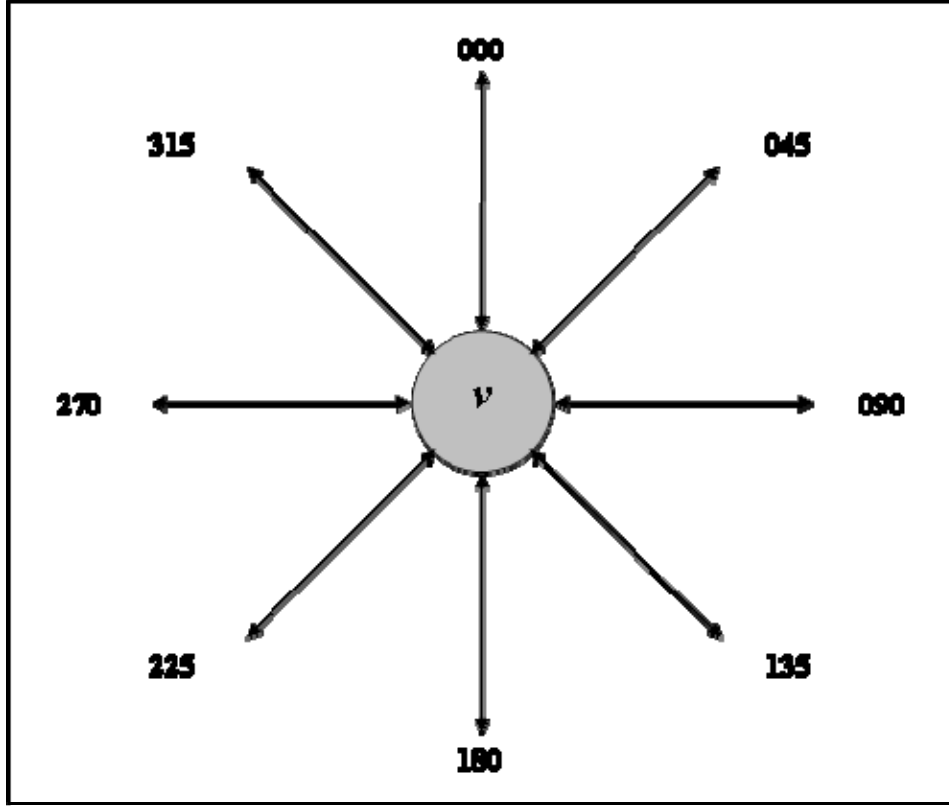
Figure 3.   Spatial adjacencies for a vertex v and associated bearings in degrees.

2.   Configurations and Direction of Travel

The positioning of escort ships around the HVU is critical for optimizing CSG defense against ASCMs. This research uses "configuration" to describe a specific escort configuration around the HVU. We assume four escort ships in the CSG, and define 16 unique configurations. The CSG's direction of travel must be taken into consideration as well. This is discussed below.

An *expanded network* $H = (N, A)$ modifies the physical network G by adding the 16 configurations, $f \in F = \{f_1, f_2, ..., f_{16}\}$ to the vertices of G, as well as eight directions of travel $d \in D = \{d_1, d_2, ..., d_8\}$. The CSG moves along a physical arc (u, v) in the direction of the arc d_{uv}. For simplicity, we assume that once the CSG reaches v, any direction d_{uv} is possible, except for $d_{uv} + 180$ degrees, i.e., directly back from where the

CSG came. A node is defined by $i = \langle v, f, d \rangle$, where v is a vertex in the physical network, f is a configuration, and d gives the direction that the CSG will travel from this vertex. An arc in H is defined by $k = (i, j) = (\langle u_i, f_i, d_i \rangle, \langle v_j, f_j, d_j \rangle)$, where (u_i, v_i) is a physical edge; d_i is the bearing that the CSG must follow while transiting (u_i, v_i); and the transition from f_i to f_j along (u_i, v_j) must be possible. In a more precise model, the transition from f_i in direction d_i to f_j in direction d_j would also be restricted.

Thus, the expanded network $H = (N, A)$ is defined by

$$N = V \times F \times D = \{i = \langle v, f, d \rangle \mid v \in V, f \in F, d \in D\}, \text{ and} \tag{3}$$

$$A = \{k = (\langle u, f, d \rangle, \langle v, f', d' \rangle) \mid (u, v) \in E, \text{ and}$$
$$\text{transitions from } f \text{ to } f' \text{ and } d \text{ to } d' \text{ on } (u, v) \text{ are feasible}\}. \tag{4}$$

H may be viewed as existing in a three-dimensional space to allow for transitions between configurations and direction combinations $\langle f, d \rangle$ in the network. Transitions are "allowable" (i.e., feasible) if they conform to certain constraints, such as an escort ship's maximum stationing speed. Stationing speed is defined as the escort's required speed to maneuver to a new position relative to the HVU within a given period of time. Stationing speed is usually faster than the CSG's normal transit speed (known as Speed of Advance). We assume that the CSG is transiting the chokepoint as fast as it can, so that escorts can only fall back in their positions relative to the HVU. Allowable transitions will be specified later, after we define configurations.

3. Sector Representation

Each configuration in this thesis is defined as a unique configuration of escorts positioned in "sectors" relative to the HVU. A sector is defined as the space between the eight compass bearings 000, 045,…, 315 degrees. The HVU is assumed to be headed "north relative," that is, 000 degrees is always relative to the direction in which the HVU is heading. This spatial representation allows each sector to be assigned a specific

13

number which remains unchanged, no matter what configuration the CSG uses in its transit path. Each sector is numbered sequentially from one to eight; see Figure 4.

In configuring escorts around an HVU in the littoral, it suffices to give positions by sector only, because the distance between the HVU and its escorts will be defined for the specific tactical requirements. While not a defined standard by any means, eight sectors should be an adequate level of granularity for locating escorts.
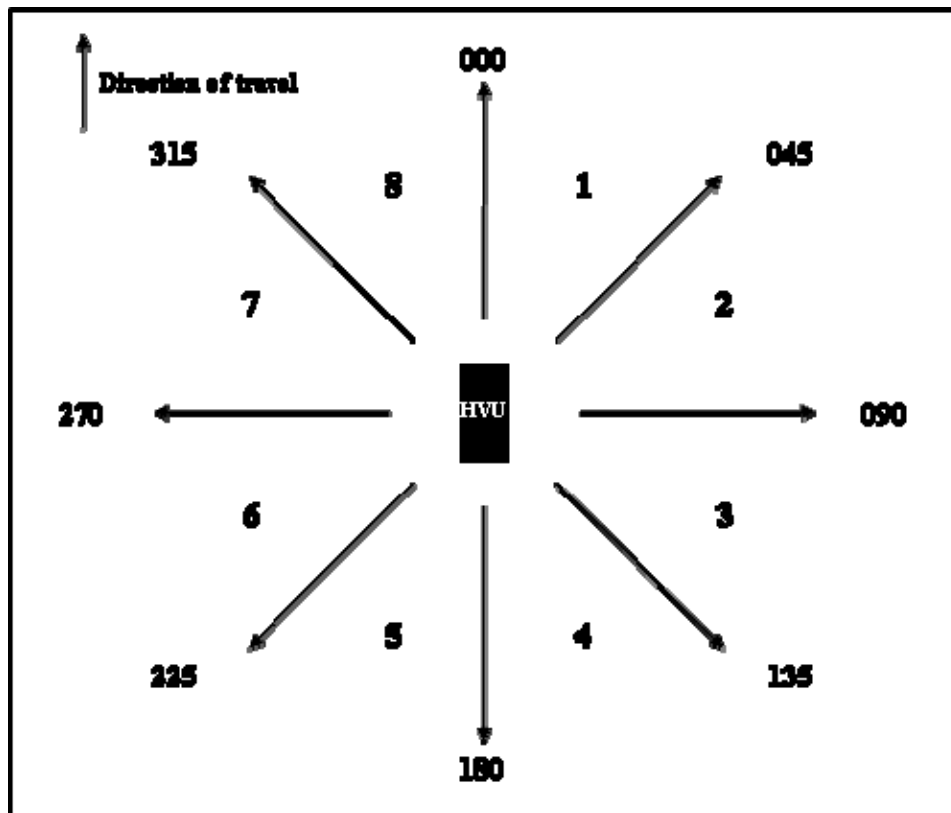
Figure 4.   Sector numbers relative to the HVU. The HVU is headed north in this figure, but the sector numbers are always relative to the direction of travel.

Table 1 defines the 16 configurations used in this thesis. Ship placement in a sector is represented as a binary choice, where "1" indicates a ship positioned in the sector, and "0" indicates no ship positioned in the sector. For example, configuration one

14

has escort ships in sectors one, two, seven, and eight; no ships are in sectors three, four, five and six. We allow at most one escort per sector, because two ships in a single sector would be unrealistically crowded.

	SECTOR							
Config.	1	2	3	4	5	6	7	8
1	1	1	0	0	0	0	1	1
2	1	1	0	0	0	1	1	0
3	1	1	0	0	1	1	0	0
4	1	1	0	0	1	0	0	1
5	1	0	0	1	0	0	1	1
6	1	0	0	1	0	1	1	0
7	1	0	0	1	1	1	0	0
8	1	0	0	1	1	0	0	1
9	0	1	1	0	0	0	1	1
10	0	1	1	0	0	1	1	0
11	0	1	1	0	1	1	0	0
12	0	1	1	0	1	0	0	1
13	0	0	1	1	0	0	1	1
14	0	0	1	1	0	1	1	0
15	0	0	1	1	1	1	0	0
16	0	0	1	1	1	0	0	1

Table 1.   Ship sector positioning for each configuration. The value for the table entry (configuration, sector) specifies whether the given sector has one ship or no ships in it for that configuration.

Table 2 shows the allowable configuration transitions, which we construct as a "transition matrix" that is used as an input to the algorithm. These transitions are represented as a binary choice as well, where "1" indicates an allowable transition, and "0" indicates otherwise. For example, a transition from configuration one to two is allowed, but a transition from two back to one is not.

		Ending Configuration at j															
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Staring Configuration at i	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	2	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
	3	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0
	4	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
	5	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
	6	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0
	7	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
	8	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0
	9	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
	13	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
	14	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0
	15	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0
	16	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1

Table 2.　　　Allowable transitions between configurations along an arc $k = (i, j)$.

C.　PROBABILITY OF SUCCESSFUL DEFENSE

The "risk" associated with an arc in the network is based on the CSG's probability of successful defense, or "Pd," associated with each arc $k = (i, j) = (<u, f, d>, \ <v, f', d'>) \in A$. In order to find a path with the highest probability of a successful transit, or "overall Pd," we define the following model, "IXRP." We note that probability formulas are meant to impart a flavor of realism, but that more exact formulas would be needed in an operational model

Indices

$i \in N$	node in expanded network
i_1	entry node in expanded network
i_m	terminal node in expanded network
$k = (i, j) \in A$	arc in expanded network
f_i	configuration at node i
d_i	direction of travel from node i
$t \in T_i$	threats visible from node i
s_{ti}	escort sector in which threat t lies at node i
r_{ti}	range from threat t to node i in nautical miles
$s \in S_{f_i}$	sectors that contain an escort at node i if the configuration is f

Data

ℓ_k	physical length of arc k in nm
$L_{\max}$	$\max_k \ell_k$
$R_{\max}$	maximum range of threat missile

Probabilities

ϕ_0 — Nominal Pd given that the HVU traverses an arc length $L_{\max}$, all within a threat zone, and without escorts; $\phi_0 \equiv 0.9$ in all tests

$\phi^S(s, s', r)$ — Pd given a single escort in sector s with a threat entering from sector s' at a range of r nautical miles, but assuming the threat remains constant as the HVU and escort travel for a distance of $L_{\max}$ nm

$\phi^C(f_i, d_i, s_{ti}, r_{ti})$ — Pd given escorts in configuration f_i traveling in direction d_i subject to a threat t entering from sector s_{ti} at a range of r_{ti} nautical miles (and assuming the threats remain constant for a distance $L_{\max}$ nm)

ϕ_i^N Pd at node i over a fictitious arc of length L_{max}, pointed in direction d_i and having configuration f_i for its full length

ϕ_k^A Pd over arc $k = (i, j)$

Probability Calculations

$$\phi^S(s, s', r) = \begin{cases} \phi_0 (1.0 - \min\{|s - s'|, 8 - |s - s'|\} & \text{if } r \le R_{max} \\ 0 & \text{if } r > R_{max} \end{cases}$$

for any allowable transition from s to s', assuming the distance   (5)
between the threat and the CSG stays at r

$$\phi^C\left(f_i, d_i, s_{ti}, r_{ti}\right) = 1 - \prod_{s' \in S_{f_i}} \left(1 - \phi^S(s, s', r_{ti})\right) \forall t, i \tag{6}$$

$$\phi_i^N = \prod_{t \in T_i} \phi^C\left(f_i, d_i, s_{ti}, r_{ti}\right) \forall i \tag{7}$$

$$\phi_k^A = ((\phi_i^N + \phi_j^N)/2)^{\ell_k/L_{max}} \quad \forall k = (i, j) \in A \tag{8}$$

Formulation for IXRP

Find a directed path $A_{Path} = \{(i_1, i_2), (i_2, i_3), ..., (i_{m-1}, i_m)\}$

such that $\prod_{k \in A_{Path}} \phi_k^A$ is maximized

The method used to calculate threat visibility resembles previous work by Leary (1995), who develops equations to compute radar ranges to arcs in a network. We consider line-of-sight calculations to be a function of threats $t \in T_i$ visible from node i. For simplicity, each such threat site $t \in T_i$ is located at a vertex in the network representing land, close to shore, in the AT. This prototypic model has only three different "elevations:" land, untraversable shoal water, and traversable water, which are described in detail in Chapter IV.

A line is established between node i and threat T_i to determine visibility. If the length of that line exceeds R_{max} (i.e., the maximum range of the threat missile in nautical miles), the threat is assumed to be "not visible." Otherwise, all intermediate nodes

between i and T_i are identified. If none of these nodes are "land nodes," then T_i is assumed visible from i; otherwise it is not. If the threat is not visible from i, then it does not come into play when computing ϕ_i^N.

In the above model, ϕ_k^A is the probability of successfully traversing an arc of length $L_{\max}$, given that the threat and configuration defined at node i remains constant along the whole arc. $L_{\max}$ is the length of the longest arc in the network, and thus $L_{\max} = \sqrt{12.5}$. For simplicity then, the probability of successfully traversing arc $k = (i, j)$ is taken as the average of ϕ_i^N and ϕ_j^N, but adjusted for the actual length of the arc.

THIS PAGE INTENTIONALLY LEFT BLANK

III. AN ALGORITHM FOR FINDING MAXIMUM-RELIABILITY PATHS FOR A CARRIER STRIKE GROUP

This chapter first describes an algorithm, XRPA, for finding maximum-reliability paths in a standard network, and then shows how to modify that algorithm for the CSG-routing problem. The modified algorithm, IXRPA, works on an implicit representation of the expanded network. We then provide pseudo-code for XRPA and IXRPA.

A. AN ALGORITHM FOR FINDING MAXIMUM-RELIABILITY PATHS IN A STANDARD NETWORK

We refer to the basic maximum-reliability path model as XRP. The XRP algorithm is referred to as "XRPA." XRPA is a label-correcting algorithm for finding all s-v maximum-reliability paths, where the reliability of a path E_{path} is defined on the physical graph $G = (V, E)$ as:

$$\prod_{(u,v) \in E_{\text{path}}} \phi_{uv} \, .$$

XRPA simply maximizes the above product, as in equation (1), rather than the sum in equation (2). The following XRPA pseudo-code essentially implements the label-correcting shortest-path algorithm using a deque, as described by Ahuja et al. (1993, pp. 136-138, 143).

Algorithm XRPA (A label-correcting algorithm for finding all s-v maximum-reliability paths)

Input: $G = (V, E)$, start vertex $s \in V$, probabilities of successful defense ϕ_k^A for all $(u, v) \in E$;

Output: Maximum probability $P(v)$ of successful transit from vertex s to all vertices $v \in V$; (The $pred()$ function encodes the "maximum-reliability tree" rooted at s, which is analogous to the shortest-path tree in a shortest-path algorithm.)

{
[1] Dummy;

[2] $P(s) \leftarrow 1$;

21

[3] $P(u) \leftarrow 0$ for all $u \in V - s$;

[4] $pred(s) \leftarrow s$;

[5] $pred(u) \leftarrow \infty$ for all $v \in V - s$;

[6]   Initialize deque Q with s;

[7]   While (Q not empty) {

[8]   Remove u from front of Q;

[9]   For (each edge of the form $(u, v) \in E$) {

[10]   If ($P(v) < P(u) \cdot p_{uv}$) {

[11]   If ($P(v) = 0$) { /* vertex v visited first time */

[12]   Put v on end of Q;

                              } else {   /* vertex v has been visited before */

[13]   If ($v \notin Q$) Put v on front of Q;

                              }

[14] $P(v) \leftarrow P(u) \cdot p_{uv}$;

[15] $pred(v) \leftarrow u$;

                       }

               }

        }

[16]   For (all $v \in V$) Print($v, P(v)$);

}

B.   IMPLICIT MAXIMUM-RELIABILITY PATH ALGORITHM (IXRPA)

This section discusses how we modify XRPA in order to implicitly represent a maximum-reliability path operating on the expanded network $H = (N, A)$. We define an "implicit maximum-reliability path algorithm," IXPRA, which adds allowable escort configurations, $f \in F$, and direction of travel, $d \in D$, to the above algorithm in addition to ϕ_k^A for all edges $(u, v) \in E$.

C. PSEUDO-CODE FOR IXRPA

Algorithm IXRPA (A label-correcting algorithm for finding all *s-v* maximum-reliability paths on an explicit representation of the expanded network H)

Input: $G = (V, E)$, start vertex $s \in V$, probabilities of successful defense ϕ_k^A for all $(u, v) \in E$; allowable escort configurations $f \in F$, allowable directions of travel $d \in D$;

Output: Maximum probability $P(i)$ of successful transit from any start node $i \in N_{start}$ (defined below) to every expanded node $i \in N$;

{
[1]   Define $N_{start} = \{< v, f, d > | v \in V_{start}, f \in F, d \in D\}$;

/* Some nodes in N_{start} may be invalid, but the algorithm will ignore these */

[2] $P(i) \leftarrow 1 \; \forall i \in N_{start}$;

[3] $P(i) \leftarrow 0 \; \forall i \in N \setminus N_{start}$;

[4] $pred(i) \leftarrow i \; \forall i \in N_{start}$;

[5] $pred(i) \leftarrow \infty \; \forall i \in N \setminus N_{start}$;

[6]   Initialize deque Q with s;

[7]   While (Q not empty) {

[8]   Remove i from front of Q;

[9]   Let u_i denote the "tail vertex" associated with i and let v_i denote the

                 head vertex associated with i

                 /* i defines a tail vertex u_i and a direction of travel, which thus defines

                 the next vertex, v_i */

[10]   If v_i is a land node, continue;

[11]   For (each allowable configuration f at v_i){

[12]   For (each allowable direction of travel d at v_i){

[13]   Let j denote the node defined by v_i, f, and d;

[14]   If ($P(j) < P(i) \cdot p_{ij}$){

23

[15]   If ($P(j) = 0$) { /* vertex i visited first time */

[16]   Put j on end of Q;

                 } else {   /* vertex i has been visited before */

[17]   If ($j \notin Q$) Put j on front of Q;

                 }

[18] $P(j) \leftarrow P(i) \cdot p_{ij}$;

[19] $pred(j) \leftarrow i$;

               }

            }

        }

    }

[20]   For(all $i \in N$) Print($i, d(i)$);

}

Note: The actual algorithm has a simple "tie-breaking mechanism" added, that allows a path to be continued to a node j if the current value of $P(j)$ is the same as the updated value will be, but the new path will use fewer "hops," i.e., arcs. This avoids the creation of paths in which the CSG makes unnecessary course corrections when under no threat.

IV. COMPUTATIONAL RESULTS

This chapter begins by introducing three test scenarios, and then describes the results of running IXRPA to solve scenario 1, "IXRPl." We then present scenarios 2 and 3, "IXRP2" and "IXRP3," respectively, and show how their results differ from IXRP1. All test runs are carried out on a desktop computer with a 3.72 GHz Intel Xeon processor, 3 gigabytes of RAM, the Microsoft Windows XP Professional operating system, and with programs written and compiled using Microsoft Visual C++ Version 6.0. No scenario requires more than one second to evaluate.

A.   TEST SCENARIOS

The test scenarios for this thesis cover the entrance to the Strait of Hormuz as shown in Figures 1 and 2. Hydrographic information for these scenarios is taken from Defense Mapping Agency nautical charts 62032 (1991) and 62510 (1999). The network area for both scenarios is bounded between latitude lines 26° and 27° North latitude and $055^{\circ} - 30'$ and $057^{\circ} - 20'$ East longitude. This creates a grid network 125 nm "wide" by 62.5 nm "high." A notional representation is shown in Figure 2. This model represents a CSG that makes a one-way transit from origin to destination, e.g., entry into the AOR, from the Gulf of Oman to the Arabian Gulf. Assuming different escort configurations may be required for an outbound transit, a separate model can determine an optimal path in the opposite direction.

The AT is plotted on a grid of vertices with origin coordinates (1,1) in the southwestern corner of the AT. For simplicity, this is the CSG's fixed destination, vertex t. Also for simplicity, the CSG starts at a fixed vertex s at grid position (36,1), in the southeastern part of the AT. The CSG may begin and end its travel in any allowable direction of travel and configuration.

Specific input values for the three test scenarios are given below:

- $nNodesX = 51$ (number of nodes in the horizontal (x) direction. Scale = 2.5 nm per node)

- $nNodesY = 26$ (number of nodes in the vertical (y) direction. Scale = 2.5 nm per node)

- ϕ_k^A data for each arc k (edge, configuration and direction of travel)

- $R_{max} = 25.0$ (maximum threat range in nm)

- $nConfigs = 16$ (number of configurations)

- $configSect$ = matrix of the 16 configuration sectors (Table 1)

- $configTrans$ = 16-by-16 matrix of allowable configuration transitions within the AT (Table 2)

- $nThreats = 5$ (in coordinate location within the network)

- Scenario threat locations (node coordinates): (7, 17), (17, 24), (27, 12), (29, 25), and (49, 12)

- Terrain Data:

   - 0.0 = unrestricted (i.e., traversable) water

   - 1.0 = untraversable shoal water

   - 5.0 = land

Several of the above parameters are "hardwired" in the code. For instance, the number of configurations, $nConfigs$, has been set to 16, and the number of threats in the network, $nThreats$, has been set to five. Each threat is assigned a coordinate position that remains fixed in the same location for all three scenarios. We change the threat range parameter $tRange$ for a specific threat location in each scenario; $tRange$ can take values of 15, 25, or 35 nautical miles. How threat range affects the CSG's maximum-reliability path will be shown below.

This thesis assigns specific sector Pd values, or $\phi^S(s, s', r)$, to each inbound threat sector and corresponding escort sector. These numbers represent Pd given a single escort

in sector s with a threat entering from sector s' at a range of r nautical miles (equation 5). For example, suppose a threat missile is fired at the CSG from a bearing that brings it into sector one. If an escort is located in sector one (i.e., sector one is occupied) then $\phi^S(s, s', r) = 0.9$. Since sectors two and eight are adjacent to the escort in sector one, $\phi^S(s, s', r) = 0.9$ in those sectors as well. $\phi^S(s, s', r) = 0.7$ in sectors three and seven, and $\phi^S(s, s', r) = 0.5$ in sectors four, five, and six, since these three sectors are immediately opposite the HVU, and farthest away from the escort's defensive capabilities.

We use $\phi^S(s, s', r)$ to compute $\phi^C(f_i, d_i, s_{ti}, r_{ti})$, or Pd given escorts in configuration f_i traveling in direction d_i, subject to a threat t entering from sector s_{ti} at a range of r_{ti} nautical miles (equation 6). Equations (5) and (6) assume that the threat remains constant as the HVU and its escorts travel for a distance of $L_{\max}$ nautical miles. Once the above probabilities have been calculated, equations (7) and (8) are used to compute the values of ϕ_k^A in the network.
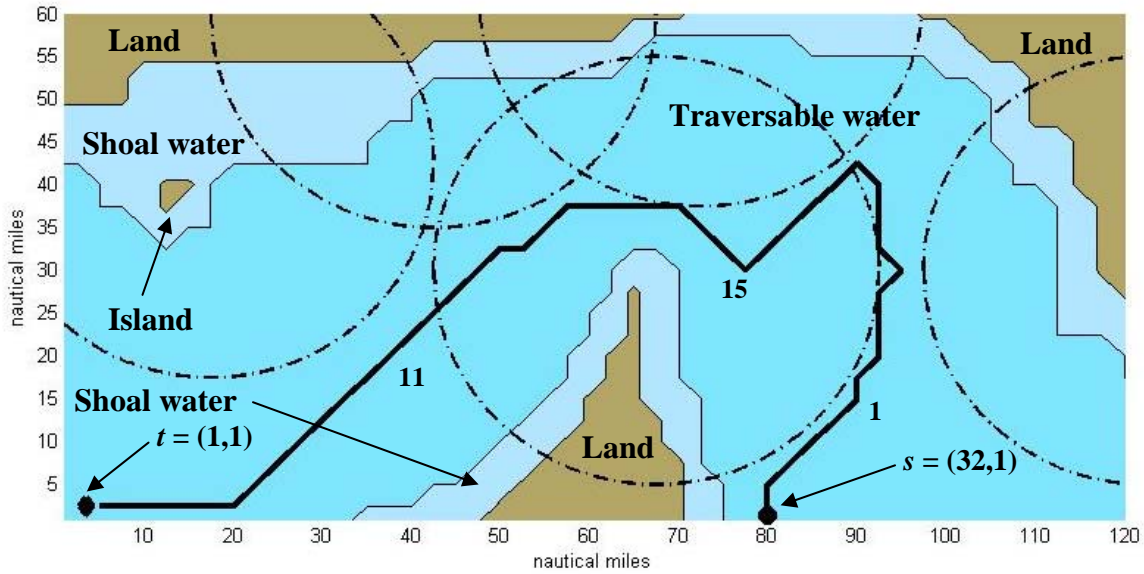
B. RESULTS FOR SCENARIO 1

Figure 5.   Maximum-reliability path for Scenario 1. The origin of this map is at the lower left-hand corner, with vertex coordinates (1,1). ASCMs are located at coordinates (7, 17), (17, 24), (27, 12), (29, 25), and (49, 12). The circles show the corresponding maximum ASCM ranges of 25 nm. At entry point $s = (32,1)$ the CSG's initial configuration is one (all four escorts are leading, ahead of the HVU's beam), traveling in direction one (north). The CSG exits at $t = (1,1)$ in configuration 11, traveling in direction one (north). Configuration numbers are shown next to the transit path. Final $\phi_k^A = 0.980407$.

Figure 5 shows the CSG's maximum-reliability path for Scenario 1.   Range circles with radius 25 nm are plotted around each threat missile site. Terrain and water-depth information is shown: traversable water is blue, with untraversable shoal water in light blue, and land in brown. Figures 5, 7 and 8 were generated using MATLAB to display the algorithm results for all three test scenarios (Chapman 2002, pp. 52-58).

The CSG uses three distinct configurations as it traverses the network in Scenario 1 (Figure 6). For example, upon entering the AT, the CSG is in Configuration 1: escorts are ahead of the HVU in sectors one, two, seven, and eight. At coordinate $(30,13)$, the CSG transitions to Configuration 15 and all escorts fall back, moving astern of the HVU into sectors three, four, five and six. Figure 6 shows the configurations in the order they

are used in Scenario 1. All transitions are reasonable: as the CSG changes course and proceeds through the AT, escort configurations change as the threat bearings change relative to the HVU.

Config. 1   Config. 15   Config. 11
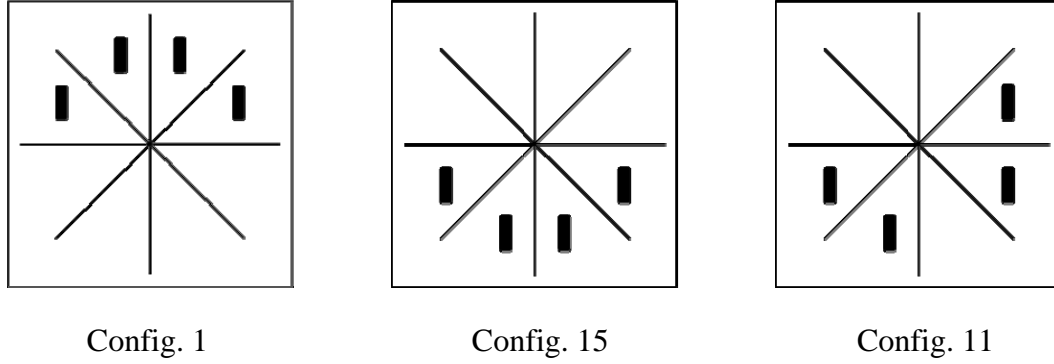
Figure 6.   Configurations in the order they are used in Scenario 1. Escort locations are shown by the black rectangles in the sectors; the HVU is situated in the center. For instance, Configuration 1 has escorts in sectors 1, 2, 7 and 8. (See Figure 4 for sector definitions.)

Table 3 shows the numerical data from IXRP1, which corresponds to the plot in Figure 5. To convert from coordinate position to actual waypoint location, multiply the x and y coordinate numbers in the table by 2.5 nm. Reading from the top left, the CSG is transiting from east to west through the AT, and enters the network at vertex $s = (32,1)$ in configuration one (all four escorts in leading positions, i.e. the four sectors forward of the HVU's beam), transiting in direction one (000 degrees, or north), and ϕ_k^A initially equal to 1.0, or a 100% chance of successful defense at entry into the AT. The CSG transits 17 more arcs before ϕ_k^A values start to decrease, because all threats are out of range until arrival at vertex $(35,16)$. The CSG exits at $t = (1,1)$ in configuration 11, transiting north, with a final probability of successful defense equal to 0.980407, meaning the CSG exits the AT with slightly more than a 98% chance of surviving the transit without damage to the HVU.

29

x-coord. (ixNode)	y-coord. (iyNode)	Config'n (iConfig)	Dir. (iDir)	Cum. ϕ_k^A	x-coord. (ixNode)	y-coord. (iyNode)	Config'n (iConfig)	Dir. (iDir)	Cum. ϕ_k^A
32	1	1	1	1.000000	23	15	15	6	0.986602
32	2	1	2	1.000000	22	14	15	6	0.985636
33	3	1	2	1.000000	21	13	15	7	0.984623
34	4	1	2	1.000000	20	13	15	6	0.983534
35	5	1	2	1.000000	19	12	15	6	0.982354
36	6	1	1	1.000000	18	11	15	6	0.981074
36	7	1	2	1.000000	17	10	11	5	0.980407
37	8	1	1	1.000000	17	9	11	5	0.980407
37	9	1	1	1.000000	17	8	11	5	0.980407
37	10	1	1	1.000000	17	7	11	5	0.980407
37	11	1	2	1.000000	17	6	11	5	0.980407
38	12	1	8	1.000000	17	5	11	6	0.980407
37	13	1	1	1.000000	16	4	11	6	0.980407
37	14	1	1	1.000000	15	3	11	6	0.980407
37	15	1	1	1.000000	14	2	11	7	0.980407
37	16	1	8	1.000000	13	2	11	7	0.980407
36	17	1	6	1.000000	12	2	11	7	0.980407
35	16	1	6	0.999326	11	2	11	7	0.980407
34	15	1	6	0.998050	10	2	11	6	0.980407
33	14	1	6	0.996908	9	1	11	7	0.980407
32	13	1	6	0.995885	8	1	11	7	0.980407
31	12	1	8	0.994963	7	1	11	7	0.980407
30	13	15	8	0.993690	6	1	11	7	0.980407
29	14	15	8	0.992464	5	1	11	7	0.980407
28	15	15	7	0.991239	4	1	11	7	0.980407
27	15	15	7	0.990010	3	1	11	7	0.980407
26	15	15	7	0.989209	2	1	11	7	0.980407
25	15	15	7	0.988386	1	1	11	1	0.980407
24	15	15	7	0.987522					

Table 3.　　　Maximum-reliability path for Scenario 1 (IXRP1). This path corresponds to the plot shown in Figure 5. Reading from the top left, the CSG enters at $s = (32,1)$ in escort configuration one, transiting in direction one (000 degrees, or north), and ϕ_k^A initially equal to 1.0. To convert x-y coordinates into waypoint locations (nm), multiply the coordinate number by 2.5.
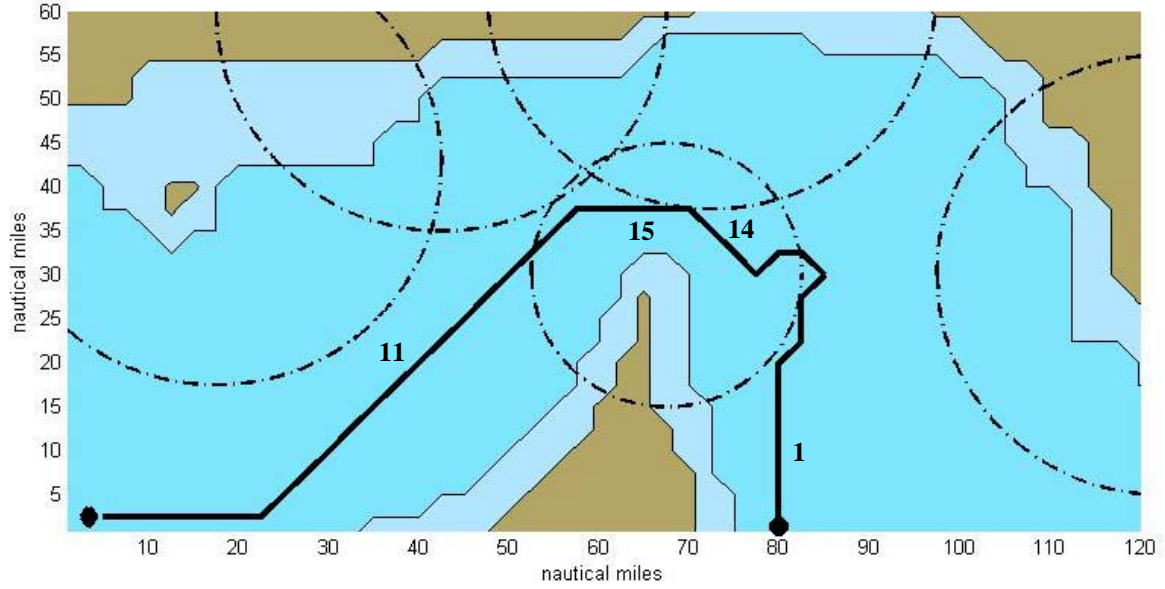
30

C. RESULTS FOR SCENARIO 2

Figure 7.   Maximum-reliability path for Scenario 2. (See Figure 5. for the labeling of the different entities in the map.) The origin of this map is at the lower left-hand corner, with vertex coordinates (1,1). ASCMs are located at coordinates (7, 17), (17, 24), (27, 12), (29, 25), and (49, 12). The circles show the corresponding maximum ASCM ranges of 25 nm, except for the ASCM located at (27, 12), whose range has been decreased to 15 nm. At entry point $s = (32,1)$ the CSG's initial configuration is one (all four escorts are leading, ahead of the HVU's beam), traveling in direction one (north). The CSG exits at $t = (1,1)$ in configuration 11, traveling in direction one (north). Configuration numbers are shown next to the transit path.

Scenario 2 (Figure 7) illustrates how the CSG's path changes when one threat missile's range is changed in the AT. Scenario 1 is set up with all threat ranges set at 25 nm. In Scenario 2, the ASCM site located on the peninsula at coordinate (27,12) decreases its range from 25 to 15 nm. Since this threat is now in range of fewer vertices in the AT, the CSG's direction of travel changes from north (Figure 5) to west (Figure 7) much sooner than in scenario 1. Consequently, the CSG's ϕ_k^A begins to decrease sooner at coordinate $(32,13)$. The path then stays closer to shoal water on the western side of

31

the peninsula, since it "escapes" the peninsula threat faster. The CSG exits at $t = (1,1)$ in configuration 11, transiting north, with a final probability of successful defense equal to 0.985893.

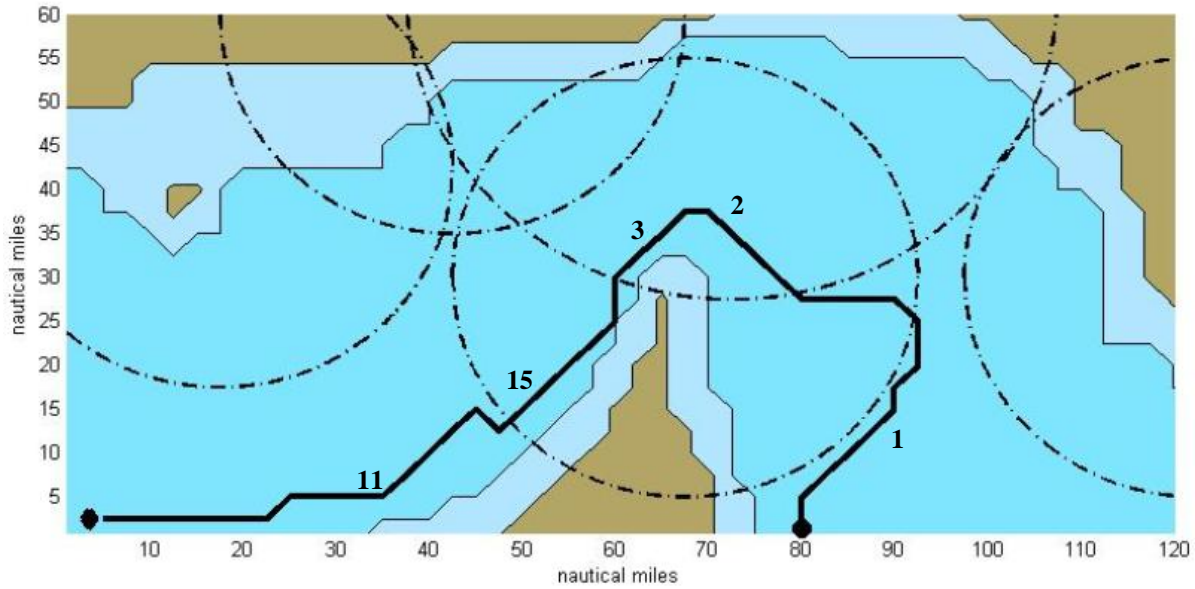## D.　　　RESULTS FOR SCENARIO 3

Figure 8.　　　Maximum-reliability path for Scenario 3. (See Figure 5. for a labeling of the different entities in the map.) The origin of this map is at the lower left-hand corner, with vertex coordinates (1,1). ASCMs are located at coordinates (7, 17), (17, 24), (27, 12), (29, 25), and (49, 12). The circles show the corresponding maximum ASCM ranges of 25 nm, except for the ASCM located at (29, 25), whose range has been increased to 35 nm. At entry point $s = (32,1)$ the CSG's initial configuration is one (all four escorts are leading, ahead of the HVU's beam), traveling in direction one (north).　　The CSG exits at $t = (1,1)$ in configuration 11, traveling in direction one (north). Configuration numbers are shown next to the transit path. (The "jog" to the northwest taken by the CSG as it leaves the last threat zone is caused by a bug in the program that has not yet been identified.)

The final scenario increases the range of ASCM site $(29, 25)$ from 25 to 35 nm. Similar to scenario 2 (Figure 7), the CSG changes course from north to west much sooner in its transit to avoid the increased range of the ASCM site to the north (Figure 8). The CSG changes to configuration 2 at coordinate $(30,13)$, which shifts the escorts on the

HVU's port (left) side back to sectors six and seven. This places the escorts more directly between the HVU and the threat to the south (i.e., the peninsula threat).

The CSG changes to configuration 3 at coordinate $(29,14)$, shifting its escorts to cover the HVU's port quarter (sectors five and six) from the peninsula threat as the CSG turns southwest. The CSG changes to configuration 15 as it continues southwest and puts the peninsula threat directly on its stern. At coordinate $(19,5)$, the CSG shifts one final time to configuration 11, which places escorts in sectors two, three, five, and six. This configuration guards the HVU's starboard (right) side from the ASCM site due north at coordinate $(7,17)$. The CSG exits at $t = (1,1)$ in configuration 11, transiting north, with a final probability of successful defense equal to 0.950729.

The table of probabilities is omitted for both scenarios 2 and 3.

E.   CONCLUSION ON COMPUTATIONAL TESTS

Comparing final ϕ_k^A values among the three scenarios illustrates the effect threat range has on the maximum-reliability path. Probability of successful defense increases from 0.980407 to 0.985893 between scenarios 1 and 2. This is reasonable, since in Scenario 2 the CSG spends less time inside the threat envelope of the ASCM on the peninsula. It is also reasonable that ϕ_k^A decreases to its lowest value, 0.950729, in scenario 3. This is the only scenario in which the CSG is exposed to multiple threats simultaneously.

The above scenarios show that based on transit geometry and threat location in the AT, the CSG relies primarily on configurations that place escorts ahead of the HVU, and always between the HVU and the threat in closest range to the CSG.

THIS PAGE INTENTIONALLY LEFT BLANK

V.   SUMMARY AND CONCLUSIONS

This thesis develops a prototype routing model that can be used to select a maximum-reliability path for a carrier strike group (CSG), or other group of ships, transiting a maritime chokepoint subject to surface-to-surface missile threats.   The solution algorithm is a variant of a shortest-path algorithm that operates implicitly on the expanded network to maximize the CSG's probability of successful defense, hence "maximizing reliability."   The model addresses the need for flexible, detailed and automated integration of data affecting position selection for Air-Defense (AD) escorts in a CSG.   The model was tested on notional data from the Strait of Hormuz, but many applications exist worldwide.

This problem is formulated and solved using a maximum-reliability path model (XRP) that identifies an optimal route for the CSG's High Value Unit (HVU) through a chokepoint, along with appropriate escort configurations for each leg of the path.   XRP discretizes the chokepoint sea space into a network of vertices and edges representing potential waypoints and transit segments between waypoints.   This network was then "expanded" to include potential waypoints, HVU escort configurations, and directions of travel.

Assuming all necessary probabilities are available, the model runs quickly for a real-world scenario based on the Strait of Hormuz (about one second on a personal computer).   The model finds an optimal path for the CSG based on the positioning of threat missile sites in the network, and as threat ranges change in the three test scenarios, so does the CSG's optimal transit path and probability of successful defense, or "ϕ_k^A."

Several simplifying assumptions are made for the sake of developing a usable model. We assume independence of attack events on an arc from the various threats, and we also assume independence of events across arcs. Given the network's relatively short arc lengths (2.5 nautical miles), these assumptions are not likely to be true, but are reasonable for a prototype. This prototype should provide the framework for a usable mission-planning aid for the fleet.

A. RECOMMENDATIONS FOR FUTURE RESEARCH

This research has developed a ship routing model based on a single type of threat, the shore-based C-802 Anti-Ship Cruise Missile (ASCM). In addition to the many different types of ASCM threats in existence today, it is well known that littoral, i.e., coastal, areas present multiple threats from enemy small boats and mines, just to name a few. Different types of threats, and their corresponding Pk data, could be added to the model (in order to keep this research unclassified, no real threat Pk data was used), and real threat range and probability of successful defense data could be used.

Escort ships in this model are considered to be indistinguishable, that is, each is equally capable of defending the HVU against a threat. This may not be realistic, however. Edge parameters represent the probability of successfully transiting between two adjacent waypoints given only one threat missile is launched per transit segment and per threat launcher, and the threat is in range of the CSG. Allowing threat missiles to fire more than once on any path leg and restricting each side to a specific number of weapons are some other possible factors to explore in future research.

Additional constraints on route selection might be developed as well, to ensure that the route selected is practical (Leary 1995). For instance, more "untransitable" vertices could be added to restrict the CSG's path to a maritime "traffic separation scheme," or terrain vertices could be decomposed into several different elevations to enable line-of-sight calculations to more closely replicate actual terrain data. Further research could also develop a more complex constrained shortest-path (CSP) network configuration that adds side constraints or expands network vertices by time or fuel consumption.

The data needed to properly route a CSG through a chokepoint do not exist today, or would take an inordinate amount of time to compute. This thesis has shown a general method is viable to identify an optimal transit path for a CSG through a maritime chokepoint once those data become available. Given the many potential threats facing today's navy in the littorals, this subject is vitally important.

LIST OF REFERENCES

Ahuja, R.K., T.L. Magnanti, and J.B. Orlin, 1993. *Network Flows Theory, Algorithms, and Applications.* Prentice Hall, Upper Saddle River, New Jersey.

Bobzin, H., 2005. *Principles of Network Economics.* Springer, Berlin.

Carlyle, W.M., J.O. Royset, and R.K. Wood, 2008. "Lagrangian Relaxation and Enumeration for Solving Constrained Shortest-Path Problems," *Networks*, to appear.

Chapman, S.J., 2002. *MATLAB Programming for Engineers* (2nd Ed.). Brooks/Cole Publishing Company, Canada.

Department of the Navy, 2005. Surface Warfare Development Group. TM 3-01.4-05, *Aegis Overland Tactics* (U).

Department of the Navy, 2006. Surface Warfare Development Group. TM 3-01.5-01, *Air Defense Ship Stationing in a Littoral Antiship Missile Environment* (U).

Defense Mapping Agency, 1991. Nautical chart 62032. *Asia: Persian Gulf.* 1:1,000,000. Defense Mapping Agency, Washington, DC.

Defense Mapping Agency, 1999. Nautical chart 62510. *Gulf of Oman – Persian Gulf: Iran – Oman – United Arab Emirates.* 1:350,000. Defense Mapping Agency, Washington, DC.

Jane's Naval Weapon Systems: Issue 44, 2006. Hooton, E.R., ed., 2006. Jane's Information Group, Limited, Surrey, United Kingdom.

Leary, John J. III, 1995. "Search for a Stealthy Flight Path through a Hostile Radar Defense Network." Master's Thesis, Operations Research Department, Naval Postgraduate School, Monterey, CA.

Montes, Anel A, 2005. "Network Shortest Path Application for Optimum Track Ship Routing." Master's Thesis, Operations Research Department, Naval Postgraduate School, Monterey, CA.

Royset, J.O. and R.K. Wood, 2007. Personal communication, 15 January.

Royset, J.O., W.M. Carlyle, and R.K. Wood, 2008. "Routing Military Aircraft with a Constrained Shortest-Path Algorithm," *Military Operations Research*, to appear.

Schildt, H., 2003. *C++ From the Ground Up* (3rd Ed). McGraw-Hill/Osborne, New York.

Sjoberg, B. and I. Will, 2006. "Optimization and Visualization of AAW System Placement and Engagement Zones," Briefing to the Office of Naval Research, 16-19 May.

Sjoberg, B., and I. Will, 2007. "Optimization and Visualization of AAW System Placement and Engagement Zones," Briefing to the Office of Naval Research, 10-11 May.

INITIAL DISTRIBUTION LIST

1.   Defense Technical Information Center
        Ft. Belvoir, VA

2.   Dudley Knox Library
        Naval Postgraduate School
        Monterey, CA

3.   Mr. Ian Will
        Naval Research Laboratory
        Washington, DC

4.   Professor R. Kevin Wood
        Naval Postgraduate School
        Monterey, CA

5.   Professor Johannes O. Royset
        Naval Postgraduate School
        Monterey, CA

6.   CAPT Jeffrey Kline, USN (Ret.)
        Naval Postgraduate School
        Monterey, CA

7.   Dr. Donald Wagner
        Office of Naval Research
        Arlington, VA